

Math 307 - Differential Equations - Spring 2017

Quiz 4

March 2, 2017

Name: Solution

Problem 1. Consider the differential equation

$$y' = \frac{10}{3}xy^{2/5}. \quad (1)$$

- (a) For which initial values do there exist solutions of (1)?
- (b) For which initial values does (1) have a unique solution?
- (c) (Extra credit) If we choose the initial value $y(0) = 0$, an easily attained solution is $y \equiv 0$. Find at least two others.

Ⓐ $f(x,y) = \frac{10}{3}xy^{2/5}$ is continuous for all x & y so solutions of (1) exist for all initial values (x_0, y_0) .

Ⓑ $f_y(x,y) = \frac{4}{3}xy^{-3/5}$ is continuous for $y \neq 0$ so solutions of (1) are unique for initial values (x_0, y_0) with $y_0 \neq 0$.

Ⓒ Separate (1): $\int y^{-2/5} dy = \int \frac{10}{3}x dx \Rightarrow \frac{5}{3}y^{3/5} = \frac{5}{3}x^2 + C$

$$\Rightarrow y^{3/5} = x^2 + C \Rightarrow y = (x^2 + C)^{5/3}$$

$$y(0) = 0: \boxed{y = x^{10/3}}$$

also

$$\boxed{y = \begin{cases} x^{10/3}, & x \leq 0 \\ 0, & x > 0 \end{cases}}$$

Piece together
constant and
non-constant
pieces.

$$\boxed{y = \begin{cases} 0, & x \leq 0 \\ x^{10/3}, & x > 0 \end{cases}}$$